Nonlinear Equivalence of Stream Ciphers

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Filter Generator

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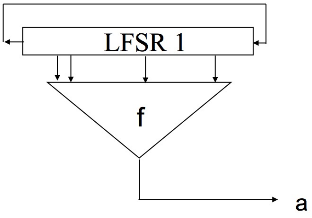
A filter generator over $\mathbb{F}_2$ is a stream cipher in perhaps its simplest form, with a well-defined mathematical description:

- it consists of a sequence generator (e.g. LFSR) and a Boolean function f, which work together to produce as output a binary string (keystream) based on the state of the register.



Filter Generator Security

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- Likewise, we know that the Hamming weight of a characteristic polynomial should not be low in order to resist correlation attacks.
- And to resist inversion attacks, the positions of the cipher's LFSR which a Boolean function taps from, should satisfy additional requirements.

However, the two components are usually analysed **separately**.

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- For example, in algebraic attacks, one collects polynomials arising from the cipher output.
- Analysis (solving) estimates then consider this set as a random set of polynomials.
- However, it has been shown that this set is very structured: for every monomial, the sequence of coefficients has a minimal polynomial which can be derived from the **LFSR** and the **Boolean function** [RH07].

Stream Cipher Equivalence

We consider (nonlinear) equivalence of LFSR-based stream ciphers using basic properties of Galois fields.

- We thus construct **isomorphism classes** of stream ciphers.
  - The topic has been studied before in the context of block ciphers (e.g. [BB02], [MR02]).

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- We thus construct **isomorphism classes** of stream ciphers.
  - The topic has been studied before in the context of block ciphers (e.g. [BB02], [MR02]).
- In our case, we show however that several important cryptographic properties, such as nonlinearity and algebraic immunity, are **not invariant** with respect to such equivalence classes.

Stream Cipher Equivalence

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Our Conclusions:

- analysis of both the generator and the corresponding Boolean function should be combined when assessing the security of a filter generator.
- furthermore, any cryptographic property should be defined with respect to the weakest equivalent cipher.
- however this seems very hard for filter generators used in practice, since the class of equivalent ciphers is very large in these cases.

Linear Feedback Shift Register

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Let **s** be the output of LFSR $\mathcal{L}$ over $\mathbb{F}_2$, with (primitive) characteristic polynomial $c(x)$ of degree n.
Let $\alpha \in \mathbb{F}_{2^n}$ be a root of $c(x)$.

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Then **s** may be written over $\mathbb{F}_{2^n}$ in terms of the roots of $c(x)$ as

$$s_t = \mathrm{Tr}(X\alpha^t) = \sum_{i=0}^{n-1} (X\alpha^t)^{2^i}, \quad t = 0, 1, 2, \ldots,$$

where the $2^n - 1$ nonzero choices of $X \in \mathbb{F}_{2^n}^*$ result in $2^n - 1$ distinct shifts of the same m-sequence **s**.

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Remark: α is a generator of $\mathbb{F}_{2^n}^*$. If β is another generator, we will use the mapping $\alpha \mapsto \beta$ to define another sequence generator.

Example

Let $n = 5, q = 2^n = 32$ and let $\mathbb{F}_2(\alpha) \simeq \mathbb{F}_{32}$, where

$$m_\alpha(x) = x^5 + x^4 + x^3 + x^2 + 1 \in \mathbb{F}_2[x]$$

is a primitive polynomial.
An m-sequence $\mathbf{s}$ can be generated as

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where $X \in \mathbb{F}_{32}^*$. Now let $\beta = \alpha^{21}$ and $X^{21} = Y \in \mathbb{F}_2(\beta)$. It follows that

$$\mathrm{Tr}(X\alpha^t) = \mathrm{Tr}((Y\beta^t)^3), t = 0, 1, 2, \ldots,$$

since $3 \cdot 21 \equiv 1 \pmod{31}$.

Example (cont.)

We can use the LFSR with characteristic polynomial $m_\beta(x)$ to generate the sequence $\mathbf{s}$.

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The corresponding sequence generator over $\mathbb{F}_2(\beta)$ is given by

$$s_t = f(b_t, b_{t+1}, \dots, b_{t+4}), t = 0, 1, 2, \dots,$$

where

$$(b_t, b_{t+1}, \dots, b_{t+4}) = (\mathrm{Tr}(Y\beta^t), \mathrm{Tr}(Y\beta^{t+1}), \dots, \mathrm{Tr}(Y\beta^{t+4})),$$

and

$$f(x_0, x_1, x_2, x_3, x_4) = x_0x_2 + x_2x_3 + x_1x_4 + x_2x_4 + x_1 + x_3.$$

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$$f(x_0, x_1, x_2, x_3, x_4) = x_0x_2 + x_2x_3 + x_1x_4 + x_2x_4 + x_1 + x_3.$$

The two filter generators (one of them is linear) will generate identical sequences for all possible initial states X and $Y = X^{21}$, and they are thus equivalent sequence generators.

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\downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \vdots & \downarrow & \downarrow \\
f(\alpha) & f(\alpha^2) & f(\alpha^3) & f(\alpha^4) & f(\alpha^5) & f(\alpha^6) & \ldots & f(\alpha^{2^n-2}) & f(\alpha^{2^n-1}) \\
\Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \Downarrow & \vdots & \Downarrow & \Downarrow \\
0 & 0 & 1 & 0 & 1 & 1 & \cdots & 0 & 1
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Note that the truth table is not complete: we do not have the image of $0 \in \mathbb{F}_{2^n}$. Thus there are equivalent functions $\overline{f}, \overline{g}$ which could also be used.

Equivalence - definitions

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Definition

For a sequence $\mathbf{s} \in \mathbb{F}_2^{q-1}$ and $\beta \in \mathbb{F}_q^*$, let

$$V_\beta(\mathbf{s}) = \{f \in \mathbb{B}_n \mid \mathbf{s} \in \mathcal{L}_\beta(f)\}.$$

We can consider $V_\beta(\mathbf{s})$ as the set of all filter generators with characteristic polynomial $g_\beta(x)$ that generate $\mathbf{s}$ as its first $q-1$ terms.

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Definition

Let $\mathbf{s} \in \mathbb{F}_2^{q-1}$ be a sequence with period e dividing $q-1$, where e is not a divisor of $2^k - 1$, with $0 < k < n$. Then let

$$\mathbb{G}_n(\mathbf{s}) = \{V_\beta(\mathbf{s}) \,|\, \beta \in \mathbb{F}_q, \;\; e \,|\, ord(\beta)\}.$$

In other words, the set $\mathbb{G}_n(\mathbf{s})$ may be viewed as a class of filter generators of length n that generate $\mathbf{s}$ as a keystream.

Number of Equivalent Filter Generators

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Lemma

Let $\mathbf{s} \in \mathbb{F}_2^{q-1}$ *denote a periodic sequence with* $e = per(\mathbf{s})$ *and* $\beta \in \mathbb{F}_q^*$ *where* $per(\mathbf{s}) \mid ord(\beta)$. *Then*

$$|V_\beta(\mathbf{s})| \leq \frac{e(q-1)}{ord(\beta)} \cdot 2^{q-ord(\beta)}.$$

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In the case of more interest ($ord(\beta) = q - 1$): we have equality and there are 2 elements in $V_\beta(\mathbf{s})$ (except for affine equivalence).

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Theorem

If $\mathbf{s} \in \mathbb{F}_2^{q-1}$ *has period* $q - 1$, *then*

$$|\mathbb{G}_n(\mathbf{s})| = \phi(q-1)/n,$$

where $\phi(q-1)$ *is the number of generators of* $\mathbb{F}_q^*$.

Another Example I

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Consider the binary sequence

$$\mathbf{s} = (1011111101000100110001010110001),$$

of length 31.

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For each (distinct) generator β of the multiplicative group of $\mathbb{F}(\alpha)$, we compute a function f_β such that $\mathbf{s} \in \mathcal{L}_\beta(f_\beta)$, where we let $g_\alpha = x^5 + x^2 + 1$.

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For each (distinct) generator β of the multiplicative group of $\mathbb{F}(\alpha)$, we compute a function f_β such that $\mathbf{s} \in \mathcal{L}_\beta(f_\beta)$, where we let $g_\alpha = x^5 + x^2 + 1$.
The distinct nonzero coset-leaders modulo 31 are $K = \{1, 3, 5, 7, 11, 15\}$, and thus we may compute six functions $f_{\alpha_k}, k \in K$, where we let $\alpha_k = \alpha^k$ and pick one function f_{α_k} from each class $V_{\alpha_k} \in \mathbb{G}_5(\mathbf{s})$.

Another Example II

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We have 6 functions $f_{\alpha_k} \in V_{\alpha_k}(\mathbf{s}) \in \mathbb{G}_5(\mathbf{s}), k \in K$:

$$
\begin{aligned}
f_{\alpha_1} &= x_0x_1x_2x_3 + x_0x_1x_2x_4 + x_0x_1x_3x_4 + x_1x_2x_3x_4 + x_0x_1x_2 + x_0x_1x_3 + \\
&\quad x_0x_2x_3 + x_1x_2x_3 + x_0x_1x_4 + x_2x_3x_4 + x_0x_2 + x_0 + x_1 \\
f_{\alpha_3} &= x_0x_1x_2x_3 + x_0x_1x_3x_4 + x_1x_2x_3x_4 + x_0x_1x_2 + x_0x_1x_4 + x_0x_3x_4 + \\
&\quad x_1x_3x_4 + x_2x_3x_4 + x_0x_1 + x_1x_3 + x_2x_4 + x_2 + x_3 \\
f_{\alpha_5} &= x_0x_1x_2x_4 + x_0x_2x_3x_4 + x_1x_2x_3x_4 + x_0x_1x_2 + x_0x_1x_3 + x_0x_2x_3 + \\
&\quad x_1x_2x_3 + x_0x_1x_4 + x_0x_3x_4 + x_0x_2 + x_0x_4 + x_1x_4 + x_2x_4 + x_0 + \\
&\quad x_1 + x_2 + x_3 + x_4 \\
f_{\alpha_7} &= x_0x_1x_3 + x_0x_2x_3 + x_1x_2x_4 + x_1x_3x_4 + x_2x_3x_4 + x_1x_2 + x_0x_3 + \\
&\quad x_0x_4 + x_1x_4 + x_3x_4 + x_0 + x_3 \\
f_{\alpha_{11}} &= x_0x_1x_2 + x_0x_2x_3 + x_1x_2x_3 + x_0x_1x_4 + x_1x_2x_4 + x_0x_1 + x_0x_2 + \\
&\quad x_1x_3 + x_0x_4 + x_2 \\
f_{\alpha_{15}} &= x_0x_1 + x_1x_2 + x_1x_3 + x_0x_4 + x_1x_4 + x_2x_4 + x_3x_4 + x_0 + x_1 + x_3
\end{aligned}
$$

Another Example III

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The columns of the table below are ordered by the 6 functions $f_{\alpha_k} \in V_{\alpha_k}(\mathbf{s}) \in \mathbb{G}_5(\mathbf{s}), k \in K$:

	f_{α_1}	f_{α_3}	f_{α_5}	f_{α_7}	$f_{\alpha_{11}}$	$f_{\alpha_{15}}$
n	5	5	5	5	5	5
d	4	4	4	3	3	2
w_H	16	16	16	16	16	16
NL	10	10	10	8	12	8
AI	2	3	2	2	3	2
CI	0	0	0	1	0	1

Note that, apart from the weight of the truth-tables and the number of variables, none of the other properties remain the same with respect to the transformations.

Cryptanalytic Implications

If we restrict ourselves to keystream-sequences of period $q - 1 = 2^n - 1$, which is the common case for sequences generated by filter generators, then there are $2 \cdot |\mathbb{G}_n(\mathbf{s})|$ isomorphic filter generators generating the same keystream sequence(s), excluding affine equivalence.

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If we restrict ourselves to keystream-sequences of period $q - 1 = 2^n - 1$, which is the common case for sequences generated by filter generators, then there are $2 \cdot |\mathbb{G}_n(\mathbf{s})|$ isomorphic filter generators generating the same keystream sequence(s), excluding affine equivalence.

Thus, in order to assess the cryptographic properties of a filter generator, one should in theory check whether there exist in this class weak isomorphic ciphers with respect to some cryptographic property.

Cryptanalytic Implications

In particular, any cryptographic property **should** be defined with respect to the weakest cipher in the equivalence class.

Definition

Let $\mathcal{P}$ be a cryptographic measurement of a filter generator $\mathcal{S}$, which generates a sequence $\mathbf{s}$. Then the filter generator $\mathcal{S}$ is said to be $\mathcal{P}$-resistant only if there is no isomorphic filter generator $\mathcal{S}'$ with measurement $\mathcal{P}' < \mathcal{P}$.

We discuss in the paper a few concepts (e.g. Algebraic Immunity), but do not extend the analysis.

Cryptanalytic Implications

The bad news: at this stage, this is likely to have limited application in practice.

- for sizes used in practice, the number of elements in the equivalence classes is huge.
- filtering functions are likely to be hard to describe (very dense with many variables).

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- filtering functions are likely to be hard to describe (very dense with many variables).

Areas for further research include:

- studying classes of Boolean functions which are equivalent with respect to both nonlinear and linear equivalence.
- generalising the idea, and defining equivalence with respect to the set of all possible combiner-generators generating a periodic sequence.

Conclusions

Given a LFSR-based stream cipher $\mathcal{S}$ generating a sequence $\mathbf{s}$, we showed how to define an equivalence class $\mathbb{G}_n(\mathbf{s})$, consisting of all filter generators of length n that produce $\mathbf{s}$ as output (and in most cases of interest, of all filter generators equivalent to $\mathcal{S}$).

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- Somewhat surprisingly, several properties of cryptographic relevance are not invariant among the elements of $\mathbb{G}_n(\mathbf{s})$.

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- Somewhat surprisingly, several properties of cryptographic relevance are not invariant among the elements of $\mathbb{G}_n(\mathbf{s})$.
- Thus, we feel that a security analysis is **incomplete** without considering the elements in $\mathbb{G}_n(\mathbf{s})$.
  - For example, one should not reach conclusions of the security properties of a filter generator by, for instance, analysing the algebraic degree or algebraic immunity of the corresponding Boolean function,
  - or the properties such as the weight of the polynomial defining the LFSR, or by the position of the registers that are tapped as input to the Boolean function.

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More research is required...

Thank you!